$$\text{Volume}=2\int_0^a \pi y^2dx=\pi\int_0^a \{a\sqrt{[a^2+8x^2]}-a^2-2x^2\}dx$$

$$=\pi\left[\frac{ax}{2}\sqrt{[a^2+8x^2]}+\frac{a^3}{4\sqrt{2}}\log\{\sqrt{[a^2+8x^2]}+2\sqrt{2}\,x\}-a^2x-\tfrac{2}{3}x^3\right]_0^a$$

$$=\frac{\pi a^3}{2}\left[\frac{1}{2\sqrt{2}}\log[3+\tfrac{1}{2}\sqrt{2}]-\tfrac{1}{3}\right]=\frac{\pi a^3}{2}\left[\frac{1}{2\sqrt{2}}\log[1+\sqrt{2}]^2-\tfrac{1}{3}\right]$$

$$=\frac{\pi a^3}{2}\left[\frac{1}{\sqrt{2}}\log[1+\sqrt{2}]-\tfrac{1}{3}\right].$$

167. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Integrate, $\displaystyle\int_0^a\int_0^b\int_0^c \frac{zdxdydz}{(x^2+y^2+z^2)^{\frac{5}{2}}}$

Solution by the PROPOSER.

$$\int_0^a\int_0^b\int_0^c \frac{zdxdydz}{\sqrt{\{(x^2+y^2+z^2)^5\}}}=\tfrac{1}{3}\int_0^a\int_0^b\left[\frac{1}{\sqrt{\{(x^2+y^2)^3\}}}\right.$$

$$\left.-\frac{1}{\sqrt{\{(c^2+x^2+y^2)^3\}}}\right]dxdy$$

$$=\frac{b}{3}\int_0^a\left[\frac{1}{x^2\sqrt{(b^2+x^2)}}-\frac{1}{(c^2+x^2)\sqrt{(b^2+c^2+x^2)}}\right]dx$$

$$=-\tfrac{1}{3}\left[\frac{\sqrt{(b^2+x^2)}}{bx}+\frac{1}{c}\tan^{-1}\left(\frac{bx}{c\sqrt{(b^2+c^2+x^2)}}\right)\right]_0^a$$

$$=-\tfrac{1}{3}\left[\frac{\sqrt{(a^2+b^2)}}{ab}+\frac{1}{c}\tan^{-1}\left(\frac{ab}{c\sqrt{(a^2+b^2+c^2)}}\right)\right]+\infty=\infty$$

DIOPHANTINE ANALYSIS.

113. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find the four least integral numbers such that the difference of every two of them shall be a square number.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and J. A. SANDERS, Hackney, Ohio.

Let a, m^2+a, n^2+a, p^2+a be the numbers.

Then $p^2+a-a=p^2$, $n^2+a-a=n^2$, $m^2+a-a=m^2$, $p^2+a-n^2-a=p^2-n^2$, $p^2+a-m^2-a=p^2-m^2$, $n^2+a-m^2-a=n^2-m^2$.